

Time : 3 Hrs.

D1G
Engg. Math.-II

Full Marks : 80

Pass Marks : 26

All question of Group-A is compulsory. Answer **any six** from Group-B, **any two** from Group-C, and **all** from Group-D.

खी -A ds l Hkh i tu vfuok; l gñ xí -B l s fclgha n% xí -C l s fclgha n%
xí -D ds l Hkh i tu dsmÚkj nA

The figures in right hand margin indicate full marks.

i k' o' l ds v d i w k e d ds l p d gñ

GROUP-A

1. Select most suitable answer from the given alternatives : **1x16=16**

fn; s x; s fodYi ka ea l s l o k e / k d mi ; Ø r m Úkj p q d j fy [ka %

(i) The domain of $f(x) = \frac{1}{x^2 - 5x + 6}$ is

$$f(x) = \frac{1}{x^2 - 5x + 6} \text{ dk i k l r g k s k \&}$$

(a) $R - \{2, 3\}$ (b) 5

(c) $\{6, 7\}$ (d) None % d k b / u g h i z

(ii) If $f(x) = f(x) + x f'(x)$, then $\int f(x).dx$ is equal to

(a) $(x+1)f(x)+k$

(b) $(x-1)f(x)+k$

(c) $x f(x)+k$

(d) None of these.

; fn $f(x) = f(x) + x f'(x)$, rC $\int f(x).dx$ cjkCj

gkxk&

(a) $(x+1)f(x)+k$

(b) $(x-1)f(x)+k$

(c) $x f(x)+k$

(d) buea l s dkbz ughA

(iii) The value of $\lim_{x \rightarrow 0} \left[\frac{\log \cos x}{x} \right]$ is equal to :

(a) ∞ (b) 1

(c) 0 (d) None of these.

$\lim_{x \rightarrow 0} \left[\frac{\log \cos x}{x} \right]$ dk eku gS%

(a) ∞ (b) 1

(c) 0 (d) buea l s dkbz ughA

(iv) If $xy = 1$, then $\frac{dy}{dx}$ is equal to :

; fn $xy = 1$, rks $\frac{dy}{dx}$ dk eku gS%

(a) xy (b) $\frac{y}{x}$

(c) $\frac{-y}{x}$ (d) $\frac{-x}{y}$

(v) The slope of the curve $y^2 = 4x$ at the point (1, 2) is:

fclnq(1, 2) ij $y^2 = 4x$ dh <ky gS&

(a) 45° (b) 90°

(c) 135° (d) 150°

(vi) The value of $\int e^{\sqrt{x}}.dx$ is equal to :

$\int e^{\sqrt{x}}.dx$ dk eku cjkCj gkxk &

(a) $2e^{\sqrt{x}} + \sqrt{x} + c$

(b) $e^{\sqrt{x}} + (\sqrt{x} + 1) + c$

(c) $2e^{\sqrt{x}} + (\sqrt{x} - 1) + c$

(d) $\frac{e^{\sqrt{x}}}{\sqrt{x}} + c$

(vii) The value of $\frac{\partial^2}{\partial x \partial y}(x^2 y^2)$ is equal to :

(a) $2(x^2 + y^2)$ (b) 0

(c) $4xy$ (d) xy

$$\frac{\partial^2}{\partial x \partial y}(x^2 y^2)$$

(a) $2(x^2 + y^2)$ (b) 0

(c) $4xy$ (d) xy

(viii) The value of $\int_0^{\infty} e^{-y} dy$ is equal to :

$$\int_0^{\infty} e^{-y} dy$$

(a) 0 (b) 1

(c) e (d) ∞

(ix) The particular solution of the differential equation $\frac{ds}{dt} = 3t^2 + 2$ when $t = 3$ and $s = 36$ is

$$\frac{ds}{dt} = 3t^2 + 2$$

(a) $t^3 + 2t + c$ (b) $t^3 + 2t + 3$

(c) $t^2 + t + 3$ (d) $t^2 + 2t + 2$

(x) Order of the differential equation

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = 0$$
 is equal to :

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = 0$$

Order of the differential equation is

(a) 3 (b) 2

(c) 0 (d) 1

(xi) Which of the following is a unit vector ?

Which of the following is a unit vector ?

(a) $\vec{i} + \vec{j}$ (b) $\vec{i} + \vec{j} + \vec{k}$

(c) $\frac{3\vec{i}}{5} + \frac{4\vec{j}}{5}$ (d) $3\vec{i} - 4\vec{j}$

- (xii) The projection of $2\vec{i} - 2\vec{j} + \vec{k}$ along $\vec{i} + \vec{j} + \vec{k}$ is:
 $2\vec{i} - 2\vec{j} + \vec{k}$ dk projection $\vec{i} + \vec{j} + \vec{k}$ dh l h/k ea
 gksk &
- (a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{3}$
 (c) $\sqrt{3}$ (d) 3
- (xiii) Two like parallel forces p and 2p are acting at a unit distance. The distance of their resultant force from p is :
 nks l tkrh; l ekulrj cy p vksj 2p bdkbz njh ij fØ; k'khy gÅ muds ifj.kkeh cy dh p l snjh gksk &
- (a) $\frac{1}{3}$ (b) $\frac{2}{3}$
 (c) $\frac{1}{2}$ (d) 3
- (xiv) If forces p, 2p, $\sqrt{3}p$ are in equilibrium and p is perpendicular to $\sqrt{3}p$, then the angle between p and 2p is :
- (a) 30° (b) 60°
 (c) 120° (d) 150°

- cy p, 2p, $\sqrt{3}p$ l rnyu eagsrFlk p yEc gks $\sqrt{3}p$ ij] rks p vksj 2p ds chp dk dksk gS%
- (a) 30° (b) 60°
 (c) 120° (d) 150°
- (xv) A particle is projected with a velocity u at 30° with the horizontal. The time to reach the maximum height is :
 , d d.k u iØx l sf{kfrt l 30° ij i{kfir fd; k tkrk gÅ egÅke Åpkbz ij igpus dk l e; gS&
- (a) $\frac{u}{g}$ (b) $\frac{u}{2g}$
 (c) $\frac{u^2}{g}$ (d) $\frac{u^2}{2g}$
- (xvi) ~~Two engines together produce an acceleration f~~ in a train. If each engine is drawing the train with force p, then the mass of the whole train is :
- (a) $\frac{p}{f}$ (b) $\frac{2p}{f}$
 (c) $\frac{p}{2f}$ (d) pf

nks batu , d l kfk , d Vu eaf os of) mRi lU d jrs gA

; fn iR; d batu Vu dks p cy l s [khp jgk gk rks i js Vu

dli ek=k gS %

(a) $\frac{p}{f}$ (b) $\frac{2p}{f}$

(c) $\frac{p}{2f}$ (d) pf

GROUP-B

2. Answer **any six** questions :

4x6=24

fdllghaNg i z uka ds mlkj na %

(a) If $x^m y^n = (x+y)^{m+n}$, find $\frac{dy}{dx}$.

; fn $x^m y^n = (x+y)^{m+n}$] rc $\frac{dy}{dx}$ fudkyA

(b) Find $\frac{dy}{dx}$ when $y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$.

$\frac{dy}{dx}$ fudkyA tc $y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$.

(c) Evaluate : $\int \left(\frac{1}{\sqrt{x+x}}\right) dx$.

$\int \left(\frac{1}{\sqrt{x+x}}\right) dx$ dk eku fudkyA

(d) Evaluate : $\int x \sec^2 x dx$.

$\int x \sec^2 x dx$ dk eku fudkyA

(e) Solve the following equation :

$$\frac{d^3 y}{dx^3} - 8y = 0.$$

fuEufyf [kr l ehdj .k dks gy dj a %

$$\frac{d^3 y}{dx^3} - 8y = 0.$$

(f) If \hat{a} and \hat{b} are unit vectors and \mathbf{q} is the angle between them, prove that $\sin \frac{\mathbf{q}}{2} = \frac{1}{2} |\hat{a} - \hat{b}|$.

; fn \hat{a} vkj \hat{b} bdkbz l fn'k gks rFkk \mathbf{q} muds chp dk dks k gks rks l kfc r dj a fd $\sin \frac{\mathbf{q}}{2} = \frac{1}{2} |\hat{a} - \hat{b}|$.

- (g) The resultant R of two forces P and Q acts at right angles to the direction of P. Show that the angle between the forces is $\cos^{-1}\left(-\frac{P}{Q}\right)$.

nks cyka P vkj Q dk i fj. kkeh R, P dh fn'kk ds l kfk l edk k ij fØ; k'khy gA fn [kyk, i fd cyka ds chp dk dks k $\cos^{-1}\left(-\frac{P}{Q}\right)$ gkskA

- (h) Prove that $v^2 = u^2 + 2fs$, where the symbol's have their usual meanings.

GROUP-C

Answer **any two** questions :

10x2=20

fdlgh nks i t uka ds mUkj na %

3. (a) Evaluate : $\lim_{x \rightarrow a} \left[\frac{x \sin a - a \sin x}{x - a} \right]$.

eku fudkya % $\lim_{x \rightarrow a} \left[\frac{x \sin a - a \sin x}{x - a} \right]$

- (b) If $f(x) = x \sin \frac{1}{x}$; when $x \neq 0$
 $= 0$; when $x = 0$.

Then test the continuity of $f(x)$ at $x = 0$.

fn $f(x) = x \sin \frac{1}{x}$; tc $x \neq 0$
 $= 0$; tc $x = 0$.

rc $f(x)$ dk $x = 0$ ij l rrk dh tkp djA

4. (a) Show that maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{1/e}$.

fn [kyk, i fd $\left(\frac{1}{x}\right)^x$ dk vf/kdre eku $e^{1/e}$ gA

- (b) A ladder is inclined to a vertical wall making an angle of 60° with it. A man is ascending the ladder at the rate of 2 m/sec. How fast is approaching the wall ?

60° dk dks k cukrh gpz, d l h-k ÅeoZ nhokj l syxh gA, d vkneh 2 ehVj@l dh nj l sml ij p<+jgk gA fdruh rst h l sog nhokj dh vkj c<+jgk gS\

P.T.O.

5. (a) State and prove Leibnitz's theorem.

Leibnitz ds l k/; dk dFku fy [kdj l kfc r djA

- (b) Prove that : $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$

$$\text{when } u = \sin^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right).$$

$$\text{l kfc r djafd \% } x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$$

$$\text{tc } u = \sin^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$$

6. (a) Evaluate **any two** of the following :

fuEufyf [kr ea l s **cdllghanks** dk eku fudkya %

(i) $\int_0^{\pi/2} \frac{dx}{1 + \cos x}$

(ii) $\int_0^{\pi/4} \tan^4 x \cdot dx$

(iii) $\int_0^a \frac{dx}{(a^2 + x^2)^{3/2}}$

- (b) Find the area enclosed between the circle

$$x^2 + y^2 = 25 \text{ and the straight line } x + y = 5.$$

oik $x^2 + y^2 = 25$ vkj l jy js [kk $x + y = 5$ dschp f?kjs

{k=Qy dks fudkya

GROUP-D

Answer **all** the questions :

5x4=20

I Hh i l uka ds mÜkj na %

7. Solve the following differential equation :

fuEufyf [kr vody l ehdj .k dk gy fudkya %

$$y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx} \quad \text{OR,} \quad \sec x \frac{dy}{dx} - y = \sin x$$

P.T.O.

8. Examine whether the following vectors are linearly dependent.

tkp djafid D; k fuEufyf[kr l fn'k , d js[kd fuHkj gS&

$$\vec{a} - 3\vec{b} + 2\vec{c}, 2\vec{a} - 4\vec{b} - \vec{c}, 3\vec{a} + 2\vec{b} - \vec{c}$$

OR,

Prove that necessary and sufficient condition for the three vectors \vec{a} , \vec{b} , \vec{c} to be co-planar is that

$$[\vec{a} \ \vec{b} \ \vec{c}] = 0.$$

fl) djafid rhu l fn'k \vec{a} , \vec{b} , \vec{c} ds, dryh; gkusdh vko'; d vksj i; klr 'kUkz gSfd &

9. Find the resultant of two unequal, unlike parallel forces.
- nksvl eku] fotkrh;] l ekukUrj cyka dk ifj. kkeh Kkr djA

OR,

O is the orthocentre of a triangle ABC. Forces P, Q, R acting along the lines OA, OB and OC are in equilibrium.

$$\text{Prove that } \frac{P}{BC} = \frac{Q}{CA} = \frac{R}{AB}.$$

f=Hkqt ABC dk yEc dhæ 0 gA OA, OB vksj OC dh vksj fØ; k'khy cy P, Q, R l rtyu ea gA

$$\text{fl) djafid } \frac{P}{BC} = \frac{Q}{CA} = \frac{R}{AB}.$$

10. A rifle bullet loses $\frac{1}{20}$ th of its velocity in passing through a plank, find how many such planks it would pass through before coming to rest, supposing the resistance of planks to be uniform.

fdl h dhæ dh xksh, d r[rsea?k] usdscn vi usox dk $\frac{1}{20}$ dk fgLI k [kksnrh gA ; fn r[rsdk vojksk l e: i gksrks#dusl sigys og, s sfdrusr[rka l sgkdj xqtjsh \

P.T.O.

OR,

A particle is projected with a velocity u at an angle α with horizontal. Find the velocity and direction of motion after a time t .

f{kr t l s α dksk cukrsgq , d d.k u ox l s i z k s i r f d ; k t k r k
g t l e ; ckn d.k dh xfr dk ox rFkk fn'kk Kkr dj

