2007 (A)

Time : 3 Hrs.

D1G

Engg. Math.-II

Full Marks : 80

Pass Marks : 26

*All question of Group-A is compulsory. Answer **any six** from Group-B, **any two** from Group-C, and **all** from Group-D.*

xi-*A* d **lHkh** i'u vfuok;Z gA xi-*B* l **fdUgh N%**] xi-*C* l **fdUgh nk]** xi-*D* d lHkh i'u d mÙkj nA

The figures in right hand margin indicate full marks.

ik'o d vd i.kkd d lpd gA

GROUP-A

1. Select most suitable answer from the given alternatives : **1x16=16**

fn;s x;s fodYik e l lokf/kd mi;Dr mÙkj pudj fy[k %

(i) The domain of $f(x)=\dfrac{1}{x^2-5x+6}$ is

$f(x)=\dfrac{1}{x^2-5x+6}$ dk ikUr gkxk &

(a) R – {2, 3} (b) 5

(c) {6, 7} (d) None ¼dkb ugh½

P.T.O.

(ii) If $f(x) = f(x) + x\ f'(x)$, then $\int f(x).dx$ is equal to

(a) $(x+1) f(x) + k$

(b) $(x-1) f(x) + k$

(c) $x f(x) + k$

(d) None of these.

;fn $f(x) = f(x) + x\ f'(x)$, rc $\int f(x).dx$ cjkcj gkxk&

(a) $(x+1) f(x) + k$

(b) $(x-1) f(x) + k$

(c) $x f(x) + k$

(d) bue l dkb ughA

(iii) The value of $\underset{x \to 0}{Lt} \left[\frac{\log \cos x}{x} \right]$ is equal to :

(a) ∞ (b) 1

(c) 0 (d) None of these.

$\underset{x \to 0}{Lt} \left[\frac{\log \cos x}{x} \right]$ dk eku g %

(a) ∞ (b) 1

(c) 0 (d) ¼bue l dkb ugh½

(iv) If $xy = 1$, then $\frac{dy}{dx}$ is equal to :

;fn $xy = 1$, rk $\frac{dy}{dx}$ dk eku g %

(a) xy (b) $\frac{y}{x}$

(c) $\frac{-y}{x}$ (d) $\frac{-x}{y}$

(v) The slope of the curve $y^2 = 4x$ at the point (1, 2) is:

fcUn (1, 2) ij oØ $y^2 = 4x$ dh <ky g &

(a) 45º (b) 90º

(c) 135º (d) 150º

(vi) The value of $\int e^{\sqrt{x}}.dx$ is equal to :

$\int e^{\sqrt{x}}.dx$ dk eku cjkcj gkxk &

(a) $2e^{\sqrt{x}} + \sqrt{x} + c$

(b) $e^{\sqrt{x}} + (\sqrt{x} + 1) + c$

(c) $2e^{\sqrt{x}}+\left(\sqrt{x}-1\right)+c$

(d) $\dfrac{e^{\sqrt{x}}}{\sqrt{x}}+c$

(vii) The value of $\dfrac{\partial^2}{\partial x \partial y}\left(x^2y^2\right)$ is equal to :

(a) $2(x^2 + y^2)$ (b) 0

(c) 4xy (d) xy

$\dfrac{\partial^2}{\partial x \partial y}\left(x^2y^2\right)$ dk eku cjkcj gkxk &

(a) $2(x^2 + y^2)$ (b) 0

(c) 4xy (d) xy

(viii) The value of $\int_0^\infty e^{-y}dy$ is equal to :

$\int_0^\infty e^{-y}dy$ dk eku g &

(a) 0 (b) 1

(c) e (d) ∞

(ix) The particular solution of the differential equation $\dfrac{ds}{dt}=3t^2+2$ when t = 3 and s = 36 is

vodyu lehdj.k $\dfrac{ds}{dt}=3t^2+2$ dk fo'k"k gy] tc t = 3 rFkk s = 36 g %

(a) $t^3 + 2t + c$ (b) $t^3 + 2t + 3$

(c) $t^2 + t + 3$ (d) $t^2 + 2t + 2$

(x) Order of the differential equation

$\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}+y=0$ is equal to :

vody lehdj.k $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}+y=0$ dk vkMj cjkcj gkxk &

(a) 3 (b) 2

(c) 0 (d) 1

(xi) Which of the following is a unit vector ?

fuEufyf[kr e dku bdkb lfn'k g \

(a) $\vec{i}+\vec{j}$ (b) $\vec{i}+\vec{j}+\vec{k}$

(c) $\dfrac{3\vec{i}}{5}+\dfrac{4\vec{j}}{5}$ (d) $3\vec{i}-4\vec{j}$

(xii) The projection of $2\vec{i} - 2\vec{j} + \vec{k}$ along $\vec{i} + \vec{j} + \vec{k}$ is:

$2\vec{i} - 2\vec{j} + \vec{k}$ dk projection $\vec{i} + \vec{j} + \vec{k}$ dh lh/k e gkxk &

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{3}$

(c) $\sqrt{3}$ (d) 3

(xiii) Two like parallel forces p and 2p are acting at a unit distance. The distance of their resultant force from p is :

nk lTkrh; lekukUrj cy p vkj 2p bdkb njh ij fØ;k'khy gA mud ifj.kkeh cy dh p l njh gkxh &

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{1}{2}$ (d) 3

(xiv) If forces p, 2p, $\sqrt{3}$ p are in equilibrium and p is perpendicular to $\sqrt{3}$ p, then the angle between p and 2p is :

cy p, 2p, $\sqrt{3}$ p lrqyu e gk rFkk p yEc gk $\sqrt{3}$ p ij] rk p vkj 2p d chp dk dk.k g \

(a) 30º (b) 60º

(c) 120º (d) 150º

(xv) A particle is projected with a velocity u at 30º with the horizontal. The time to reach the maximum height is :

,d d.k u iox l f{kfrt l 30º ij i{kfir fd;k tkrk gA egÙke Åpkb ij igpu dk le; g &

(a) $\frac{u}{g}$ (b) $\frac{u}{2g}$

(c) $\frac{u^2}{g}$ (d) $\frac{u^2}{2g}$

(xvi) **Two engines together produce an acceleration** f in a train. If each engine is drawing the train with force p, then the mass of the whole train is :

(a) $\frac{p}{f}$ (b) $\frac{2p}{f}$

(c) $\frac{p}{2f}$ (d) pf

P.T.O.

nk btu ,d lkFk ,d Vu e f oxof) mRiUu djrk gA

;fn iR;d btu Vu dk p cy l [khp jgk gk] rk ij Vu

dh ek=k g\\&

(a) $\frac{p}{f}$ (b) $\frac{2p}{f}$

(c) $\frac{p}{2f}$ (d) pf

GROUP-B

2. Answer ***any six*** questions : **4x6=24**

fdUgh Ng i'uk d mÙkj n \\

(a) If $x^m y^n = (x+y)^{m+n}$, find $\frac{dy}{dx}$.

;fn $x^m y^n = (x+y)^{m+n}$] rc $\frac{dy}{dx}$ fudkyA

(b) Find $\frac{dy}{dx}$ when y = $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$.

$\frac{dy}{dx}$ fudky tc y = $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$.

(c) Evaluate : $\int \left(\frac{1}{\sqrt{x}+x}\right) dx$.

$\int \left(\frac{1}{\sqrt{x}+x}\right) dx$ dk eku fudkyA

(d) Evaluate : $\int x \sec^2 x.dx$.

$\int x \sec^2 x.dx$ dk eku fudkyA

(e) Solve the following equation :

$$\frac{d^3y}{dx^3} - 8y = 0.$$

fuEufyf[kr lehdj.k dk gy dj \\

$$\frac{d^3y}{dx^3} - 8y = 0.$$

(f) If $\hat{a}$ and $\hat{b}$ are unit vectors and q is the angle between them, prove that $\sin\frac{q}{2} = \frac{1}{2}\left|\hat{a}-\hat{b}\right|$.

;fn $\hat{a}$ vkj $\hat{b}$ bdkb lfn'k gk rFkk q mud chp dk dk.k gk rk lkfcr dj fd $\sin\frac{q}{2} = \frac{1}{2}\left|\hat{a}-\hat{b}\right|$.

(g) The resultant R of two forces P and Q acts at right angles to the direction of P. Show that the angle between the forces is $\cos^{-1}\left(-\frac{P}{Q}\right)$.

nk cyk P vkj Q dk ifj.kkeh R, P dh fn'kk d lkFk ledk.k ij fØ;k'khy gA fn[kyk, fd cyk d chp dk dk.k $\cos^{-1}\left(-\frac{P}{Q}\right)$ gkxkA

(h) Prove that $v^2 = u^2 + 2fs$, where the symbol's have their usual meanings.

GROUP-C

Answer ***any two*** questions : **10x2=20**

fdUgh nk i'uk d mÙkj n

3. (a) Evaluate : $\lim\limits_{x \to a}\left[\frac{x\sin a - a\sin x}{x - a}\right]$.

eku fudky $\lim\limits_{x \to a}\left[\frac{x\sin a - a\sin x}{x - a}\right]$

(b) If $f(x) = x\sin\frac{1}{x}$; when $x \neq 0$

$= 0$; when $x = 0$.

Then test the continuity of $f(x)$ at $x = 0$.

;fn $f(x) = x\sin\frac{1}{x}$; tc $x \neq 0$

$= 0$; tc $x = 0$.

rc $f(x)$ dk $x = 0$ ij lrrk dh tkp djA

4. (a) Show that maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{1/e}$.

fn[kyk, fd $\left(\frac{1}{x}\right)^x$ dk vf/kdre eku $e^{1/e}$ gA

(b) A ladder is inclined to a vertical wall making an angle of 60° with it. A man is ascending the ladder at the rate of 2 m/sec. How fast is approaching the wall ?

60° dk dk.k cukrh gbZ ,d lh<h Åèo nhokj l yxh gA ,d vkneh 2 ehVj@lS dh nj l mlij p< jgk gA fdruh rth l og nhokj dh vkj c< jgk g\

5. (a) State and prove Leibnitz's theorem.

Leibnitz d lk/; dk dFku fy[kdj lkfcr djA

(b) Prove that : $x\frac{\partial u}{\partial x}+y\frac{\partial u}{dy}=\frac{1}{2}\tan u$

when u = $\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$.

lkfcr dj fd % $x\frac{\partial u}{\partial x}+y\frac{\partial u}{dy}=\frac{1}{2}\tan u$

tc u = $\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

6. (a) Evaluate ***any two*** of the following :

fuEufyf[kr esa ls ***fdUgha nks*** dk eku fudkys %

(i) $\int_0^{p/2}\frac{dx}{1+\cos x}$

(ii) $\int_0^{p/4}\tan^4 x.dx$

(iii) $\int_0^{a}\frac{dx}{\left(a^2+x^2\right)^{3/2}}$

(b) Find the area enclosed between the circle $x^2+y^2=25$ and the straight line x + y = 5.

oÙk $x^2+y^2=25$ vkSj ljy js[kk x + y = 5 dss chp f?kjs {ks=Qy dk fudkyA

GROUP-D

Answer ***all*** the questions : **5x4=20**

lHkh iz'uksa ds mÙkj nsa %

7. Solve the following differential equation :

fuEufyf[kr vody lehdj.k dk gy fudkys %

$y^2+x^2\frac{dy}{dx}=xy\frac{dy}{dx}$ **OR,** $\sec x\frac{dy}{dx}-y=\sin x$

8. Examine whether the following vectors are linearly dependent.

tkp dj fd D;k fuEufyf[kr lfn'k ,d jf[kd fuHkj g &

$\vec{a} - 3\vec{b} + 2\vec{c}$, $2\vec{a} - 4\vec{b} - \vec{c}$, $3\vec{a} + 2\vec{b} - \vec{c}$

OR,

Prove that necessary and sufficient condition for the three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ to be co-planar is that

$$\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right] = 0 .$$

fl) dj fd rhu lfn'k $\vec{a}$, $\vec{b}$, $\vec{c}$ d ,dryh; gku dh vko';d vkj i;kIr 'kÙk g fd &

9. Find the resultant of two unequal, unlike parallel forces.

nk vleku] fo tkrh;] lekukUrj cyk dk ifj.kkeh Kkr djA

OR,

0 is the orthocentre of a triangle ABC. Forces P, Q, R acting along the lines OA, OB and OC are in equilibrium.

Prove that $\dfrac{P}{BC} = \dfrac{Q}{CA} = \dfrac{R}{AB}$.

f=Hkt ABC dk yEc dUæ 0 gA OA, OB vkj OC dh vkj fØ;k'khy cy P, Q, R lryu e gA

fl) dj fd $\dfrac{P}{BC} = \dfrac{Q}{CA} = \dfrac{R}{AB}$.

10. A rifle bullet loses $\dfrac{1}{20}$ th of its velocity in passing through a plank, find how many such planks it would pass through before coming to rest, supposing the resistance of planks to be uniform.

fdlh cUnd dh xkyh ,d r[r e ?klu d ckn viu ox dk $\dfrac{1}{20}$ ok fgLlk [kk nrh gA ;fn r[r dk vojk/k le:i gk rk #du l igy og ,l fdru r[rk l gkdj xtjxh \

P.T.O.

OR,

A particle is projected with a velocity u at an angle a with horizontal. Find the velocity and direction of motion after a time t.

f{kfrt l a dk.k cukr g, ,d d.k u ox l i{kfir fd;k tkrk gA t le; ckn d.k dh xfr dk ox rFkk fn'kk Kkr djA

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