2007 (A)

00101

Time : 3 Hrs.

D1G

Engg. Math.-I

Full Marks : 80

Pass Marks : 26

Answer all questions in your own words.

lHkh ç'uk d mÙkj viu 'kCnk e nA

The figures in right hand margin indicate full marks.

ik'o d vd i.kk d lpd gA

English Version

1. Answer ***all*** questions : **1x20=20**

(i) The set of wires through which data or address is transferred from one unit to other unit of a computer system is called **..... .**

(ii) If ${}^{n}c_{18} = {}^{n}C_{12}$, then ${}^{32}C_{n}$ = **..... .**

(iii) Value of e is :

(a) less than 2 (b) equal to 2

(c) less than 3 (d) greater than 3.

(iv) The 11th term from the end in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{12}$ is ..... .

(v) The value of the determinant :

$$\begin{vmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 3 & 7 \\ 6 & 3 & 9 & 15 \\ 1 & 2 & 3 & 4 \end{vmatrix}$$ is ..... .

(vi) Define unit matrix.

(vii) Modulus of $(1 + 2i)(1 + 3i)(3 + 4i)$ is ..... .

(viii) If $\frac{5+xi}{2} = 2\frac{1}{2}$, find the value of x if $i = \sqrt{-1}$.

(ix) In a Δ ABC, if a = 2, b = 3 and sin A = $\frac{2}{3}$, find $\angle$ B.

(x) The minimum value of $\tan^2 q + \cot^2 q$ is .....

(xi) The value of $\sin 105^\circ + \cos 105^\circ$ is ..... .

(xii) In a Δ ABC, a = 18, b = 24 and c = 30, the value of sin A is ..... .

(xiii) In which quadrant the point (1, –9) lies ..... .

(xiv) The cartesian co-ordinate of a point is (1, 1), find its polar co-ordinate.

(xv) The length of perpendicular from origin to the line $x + \sqrt{3}y = 4$ is ..... .

(xvi) The direction cosine of the line joining p (4, 5, 0) and Q (2, 6, 2) is ..... .

(xvii) If the plane 3x + y + 4z + 2 = 0 is parallel to the plane 6x + ky + 8z = 5, the value of k is ..... .

(xviii) Equation of line perpendicular to the plane $2x + \sqrt{3}y + 3z = 5$ and passing through (1, 2, –1) is ..... .

(xix) The value of $\log_3^2 \times \log_5^3 \times \log_2^5$ is ..... .

(xx) Write full form of 'BASIC'.

P.T.O.

2. Resolve into partial fraction : **4**

$$\frac{4x^2+5}{(x-2)(x+1)(2x-3)} \quad \textbf{OR,} \quad \frac{x^4}{(x-1)^4(x+1)}$$

3. The sum of an infinite geometric series is 15 and the sum of squares of these terms is 45. Find the series. **4**

OR,

If the p^{th}, q^{th} and r^{th} term of a Harmonic progression is a, b, c respectively, then prove that

$bc\,(q-r) + ca\,(r-p) + ab\,(p-q) = 0$

4. State 'Fundamental principle of counting'. How many numbers are there between 100 and 1000 which have exactly one of their digits as 7. **4**

OR,

Define a matrix. What is difference between a matrix and determinant ? Explain with example.

5. If $\cot^2 q + \frac{3}{\sin q} + 3 = 0$, find general value of q. **4**

OR,

Prove that $2\tan^{-1} x = \sin^{-1}\left(\frac{2x}{1-x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$.

6. Find the equation of a circle which touches y-axis at a distance 4 from origin and cuts an intercept of length 6 from x-axis. **4**

OR,

Find the equation of a circle whose centre is at (h_1, k) and radius is a.

7. (a) In ΔABC prove that : **4+4=8**

$$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$$

(b) If $A + B + C = p$, prove that

$$\frac{\cot A+\cot B}{\tan A+\tan B} + \frac{\cot B+\cot C}{\tan B+\tan C} + \frac{\cot C+\cot A}{\tan C+\tan A} = 1$$

OR,

(a) If a^2, b^2, c^2 are in AP, prove that cot A, cot B, cot C are in A.P.

P.T.O.

(b) Prove that $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$

8. (a) If z_1 and z_2 are two complex numbers **4+4=8**

such that $|z_1 + z_2| = |z_1 - z_2|$

prove that : $\text{Arg}(z_1) - \text{Arg}(z_2) = \frac{p}{2}$

(b) Find the value of $\sqrt{-i}$ where $i = \sqrt{-1}$.

OR,

(a) Prove that $\left(\frac{-1+i\sqrt{3}}{2}\right)^n + \left(\frac{-1-i\sqrt{3}}{2}\right)^n$ is equal to :

(i) 2 if n is multiple of 3.

(ii) –1 if n is not multiple of 3.

(b) Find the smallest positive integer n for which :

$\left(\frac{1+i}{1-i}\right)^n = 1$.

9. (a) If $|x| < 1$ prove that : **4+4=8**

$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4}$ .....

(b) Find the co-efficient of x^{10} in the expansion of $\frac{1+3x^2}{(1-x^2)^3}$, mentioning the condition under which the result holds.

OR,

(a) Sum the series :

$1 + \frac{3}{\lfloor 2} + \frac{6}{\lfloor 3} + \frac{10}{\lfloor 4} + \ldots\ldots\ldots\ldots\infty$

(b) Find the middle term in the expansion of $\left(2a - \frac{a^2}{4}\right)^9$.

10. (a) Find the co-ordinates of a point whose distance from (3, 5) is 5 units and from (0, 1) is 10 units. **4+4=8**

(b) Obtain the equation of plane passing through the line of intersection of planes :

$7x - 4y + 7z = 16$ and $4x + 3y - 2z = 0$

and perpendicular to the plane $x - y - 2z + 5 = 0$.

P.T.O.

OR,

(a) Find the angle between the lines

$y = m_1x + c_1$ and $y = m_2x + c_2$.

Also obtain condition for perpendicularity and parallelism.

(b) Show that the joins of the point (0, 2, –1), (1, 0, 0) and (1, 2, –2), (3, –1, 1) intersect.

11. (a) Using 1's complement method find the value of $(D5F.4A)_{16} - (7531.152)_8$ as a decimal number. **4+4=8**

(b) Draw a flow-chart to find maximum of three given numbers.

OR,

(a) Convert 5745.32 and $(5742.163)_8$ into a hexadecimal number.

(b) Write a BASIC program to find sum and product of three given numbers.

Hindi Version

¼fgUnh :ikUrj½

1. /Hkh i'uk d mÙkj n % **20x1=20**

(i) dEI;Vj d rkjk dk leg] ft l d }kjk MkVk ;k irk dk dEI;Vj d ,d Hkkx l nlj Hkkx e Hkt k tkrk g] ml dgk tkrk g .....A

(ii) ;fn ${}^nC_{18} = {}^nC_{12}$, rk ${}^{32}C_n$ = ...

(iii) e dk eku g :

(a) 2 l de (b) 2 d cjkcj

(c) 3 l de (d) 3 l T;knkA

(iv) $\left(2x^2 - \frac{1}{x}\right)^{12}$ d foLrkj e vUr l 11o in dk eku gkxk ..... .

(v) lkjf.kdk

2	1	3	5
4	2	3	7
6	3	9	15
1	2	3	4

dk eku gkxk ..... .

P.T.O.

(vi) bdkb v0;g dh ifjHkk"kk fy[kA

(vii) $(1 + 2i)\,(1 + 3i)\,(3 + 4i)$ dk ekikd gkxk .....

(viii) ;fn $\frac{5+xi}{2} = 2\frac{1}{2}$ gk rk] x dk eku fudkyA ;fn $i = \sqrt{-1}$

(ix) ;fn ΔABC e a = 2, b = 3 vkj $\sin A = \frac{2}{3}$, rk $\angle B$ dk eku fudkyA

(x) $\tan^2 q + \cot^2 q$ dk U;ure eku gkxk .....A

(xi) $\sin 105^\circ + \cos 105^\circ$ dk eku gkxk ..... A

(xii) ΔABC e ;fn a = 18, b = 24 vkj c = 30 rk sin A dk eku gkxk ..... A

(xiii) fcUn (1, –9) fdl ikn e vofLFkr g \

(xiv) fdlh fcUn dk dkrh; fu;ked (1, 1) g rk bldk ikyj fu;ked ikIr djA

(xv) ey fcUn l ljy j[kk $x + \sqrt{3}y = 4$ ij yEc dh yEckb dk eku gkxk .....A

(xvi) fcUn p (4, 5, 0) vkj Q (2, 6, 2) dk feyku okyh ljy j[kk dk d.c. gkxk ..... A

(xvii) ;fn ry 3x + y + 4z + 2 = 0, ry 6x + ky + 8z = 5, d lekukUrj g rk k dk eku gkxk .....

(xviii) ljy j[kk tk ry $2x + \sqrt{3}y + 3z = 5$ ij yEc g rFkk fcUn (1, 2, –1) x tjrk g] dk lehdj.k gkxk .....A

(xix) $\log_3^2 \times \log_5^3 \times \log_2^5$ dk eku g ..... A

(xx) 'BASIC' dk foLrkj fy[kA

2. vkf'kd fHkUu d :i e 0;Dr dj & **4**

$$\frac{4x^2+5}{(x-2)(x+1)(2x-3)} \quad \textbf{vFkok]} \quad \frac{x^4}{(x-1)^4(x+1)}$$

3. fdlh vuUr x.kkUkj J.kh d ink dk ;kx 15 g vkj bu ink d oxk dk ;kx 45 gA J.kh Kkr djA **4**

vFkok]

;fn gjkRed J.kh d pok, qok vkj rok in Øe'kh a, b, c gk rk lkfcr dj fd bc (q – r) + ca (r – p) + ab (p – q) = 0.

P.T.O.

4. 'Fundamental principle of counting' dk fy[kA 100 vkj 1000 d chp fdruh ,slh l[;k, g ftuesa fIQ ,d gh 7 gA **4**

vFkok]

v0;g dh ifjHkk"kk fy[kA v0;g vkj lkjf.kdk e D;k vUrj g \ mnkgj.k ndj 0;k[;k djA

5. ;fn $\cot^2 q + \frac{3}{\sin q} + 3 = 0$, rk q dk 0;kid eku fudkyA **4**

vFkok]

lkfcr dj fd &

$$2\tan^{-1} x = \sin^{-1}\left(\frac{2x}{1-x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right).$$

6. ml oÙk dk lehdj.k fudkyo tks y-v{k dk ey fcUnq l 4 dh njh Li'k djrk g vkj x-v{k ij 6 dk vUr%[k.M dkVrk gA **4**

vFkok]

ml oÙk dk lehdj.k Kkr djk ft ldk dUæ (h_1, k) g vkj f=T;k a gA

7. (a) ΔABC e lkfcr dj & **4+4=8**

$$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$$

(b) ;fn A + B + C = p rk lkfcr djk &

$$\frac{\cot A+\cot B}{\tan A+\tan B} + \frac{\cot B+\cot C}{\tan B+\tan C} + \frac{\cot C+\cot A}{\tan C+\tan A} = 1$$

vFkok]

(a) ;fn a^2, b^2, c^2 lekUrj J.kh e gk rk lkfcr dj cot A, cot B, cot C lekUrj J.kh e gA

(b) lkfcr djk &

$\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$

8. (a) ;fn z_1, z_2 nk ,slh feJ l[;k, g fd % **4+4=8**

$$|z_1 + z_2| = |z_1 - z_2|$$

lkfcr dj $\text{Arg}(z_1) - \text{Arg}(z_2) = \frac{p}{2}$

(b) $\sqrt{-i}$ dk eku fudkyo tgk $i = \sqrt{-1}$ gA

P.T.O.

vFkok]

(a) lkfcr djk $\left(\frac{-1+i\sqrt{3}}{2}\right)^n+\left(\frac{-1-i\sqrt{3}}{2}\right)^n$ cjkcj g&

(i) 2 ;fn n rhu dk x.kd gA

(ii) –1 ;fn n rhu dk x.kd ugh gA

(b) n dk U;ure eku fudky] ft ld fy,

$\left(\frac{1+i}{1-i}\right)^n=1$.

9. (a) ;fn $|x|<1$ lkfcr dj & **4+4=8**

$\log(1-x) = -x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}$ .....

(b) $\frac{1+3x^2}{(1-x^2)^3}$ d foLrkj e x^{10} dk x.kd fudky] vkj ;g Hkh crk, fd ;g ifj.kke fdl fLFkfr e lgh gA

vFkok]

(a) fuEufyf[kr J.kh dk ;kx fudky :

$1+\frac{3}{\lfloor 2}+\frac{6}{\lfloor 3}+\frac{10}{\lfloor 4}+\ldots\ldots\ldots\ldots\infty$

(b) d foLrkj e eè; in fudkyA

10. (a) ml fcUn dk fu;ked fudky tk (3, 5) l 5 bdkb rFkk (0, 1) l 10 bdkb dh njh ij gA **4+4=8**

(b) ml ry dk leh dj.k fudky ry $7x - 4y + 7z = 16$ rFkk $4x + 3y – 2z = 0$ dh dVku j[kk l xtjrk g vkj ry $x – y –2z + 5 = 0$ ij yEc gA

vFkok]

(a) ljy j[kk, & $y = m_1x + c_1$ vkj $y = m_2x + c_2$ d chp dk dk.k fudkyA nkuk ljy j[kkvk dk yEc rFkk lekukUrj gku d fy, ifrc/k Hkh Kkr djA

(b) fn[kk, fd fcUn (0, 2, –1), (1, 0, 0) vkj (1, 2, –2), (3, –1, 1) dk feykuokyh ljy j[kk, ,d nlj dk foHkDr djrh gA

11. (a) bdkb ijd fof/k dk i;kx dj

$(D5F.4A)_{16} – (7531.152)_8$

nl vk/kkjh l[;k d :i e eku fudkyA **4+4=8**

P.T.O.

(b) rhu nh gb l[;kvk e l vf/kdre l[;k Kkr dju d fy, flow chart r;kj djA

vFkok]

(a) l[;kvk 5745.32 rFkk $(5742.163)_8$ dk 16 vk/kkjh l[;k e cnyA

(b) rhu nh gb l[;kvk dk tkMu rFkk x.kk dju d fy, 'BASIC' ikxke fy[kA

❋ ❋ ❋